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Coaxial line methods for measuring permittivity and dielectric loss

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Two coaxial line techniques for the determination of complex permittivities of solid and liquids are described. The first, the matched termination method, is essentially a comparison technique using air as the reference dielectric, producing accurate values of ϵ'. In the second, the resonant line method, the characteristic impedance termination is replaced by an adjustable short circuit. This method was developed primarily for the purpose of determining the values of tan δ in low loss materials. Both methods can be used for frequencies in the 200 MHz to 9 GHz range and normally require only conventional apparatus. The results obtained for the materials under test agreed well with the published data, thus underlining the suitability of the two techniques for dielectric measurements.

1. Introduction

Two coaxial line techniques for the determination of complex permittivities of solids and liquids are described. Both methods use a coaxial sample holder especially designed for this purpose, incorporating a parallel plate capacitor in series with the inner conductor.

The first, the matched termination method, is essentially a comparison technique using air as the reference dielectric. In this case, the changes in the voltage standing wave pattern are recorded when the sample material is inserted. Precise measurements of the reflection coefficient and shift in the standing wave minimum are of importance, as the following theory will show. In order to improve further the accuracy of measuring the voltage standing wave ratio, VSWR, its values are lowered by terminating the sample holder with the characteristic impedance, Z_0.

In the second method, the resonant line method, the characteristic impedance termination is replaced by an adjustable short circuit known as the reactive stub. After the test material is placed in the sample holder the VSWR readings are taken and plotted against the varying stub lengths. Maximum occurs at resonance when the termination of the line is resistive and this value of VSWR, VSWR_{MAX}, is related to the loss tangent of the material, i.e. $\tan\delta = \epsilon''/\epsilon'$. The procedure can be successfully employed for low loss materials when the value of ϵ' is known; possibly determined by the former matched-termination technique.

Both methods can be used for frequencies in the 200 MHz to 9 GHz range and normally require only conventional apparatus. It will be apparent from the following paragraphs, however, that better results are achievable with more sensitive instruments and higher precision components.

2. Coaxial line sample holder

The external and cross-sectional views of the sample holder are shown in Fig. 1a and b. It was made from brass with the internal and external diameters chosen to give theoretically a characteristic impedance, Z_0, of 50 ohms. The design was based dimensionally on the connector assembly of the General Radio slotted line system used in the measurements. Part of the inner conductor was made replaceable both to allow different sample thicknesses to be inserted and for possible spring removal. The access to the gap is through a cut-out window on the outer conductor. When the sample holder is in use a slide-on close-fitting ring clamps the window cover in place. A functional representation of the gap is given in Fig. 1c.

The performance of the sample holder was

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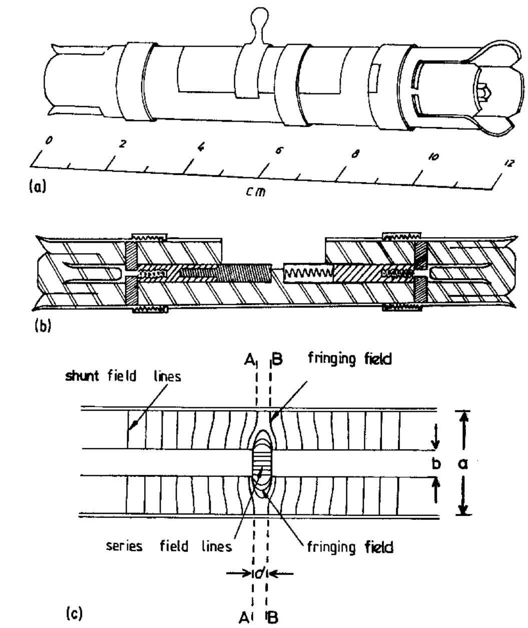


Figure 1 (a) External view and (b) cross-section of sample holder. (c) Electric field distribution.

examined by measuring its VSWR over the operational frequency range when terminated with Z_0. The resulting plot in Fig. 2a for the unit with the gap closed satisfied the requirements adequately. Further, the deviation of VSWR from the calculated values as a function of the air gap spacing in Fig. 2b gave an additional check on the possible errors introduced by the sample holder. These measurements helped in estimating the tolerances in the values of the reflection coefficient and later the accuracy of ϵ' and ϵ'' components of the complex permittivity constant. Such plots would normally be required at the frequencies of interest and for each sample holder design. Calibration of sample holders does not need any additional equipment nor different procedures from those used in the actual dielectric measurements. The block diagram in Fig. 3 shows the experimental apparatus and arrangement for the two methods.

3. The matched termination method

3.1. Theory

The sample holder with the material under test placed in the gap may be represented by the equivalent circuit of Fig. 4. The series capacitance is given by $C_0\epsilon_r^*$, where ϵ_r^* is the relative complex permittivity of the material, $\epsilon' - j\epsilon''$, and C_0 (in farads) is a function of the air permittivity, ϵ_0, and the ratio of the cross-sectional area of the inner conductor, A, and the gap spacing, d, i.e. $\epsilon_0 A/d$. The fringing field capacitance, C_f, is associated with the inner conductor discontinuities [1, 2].

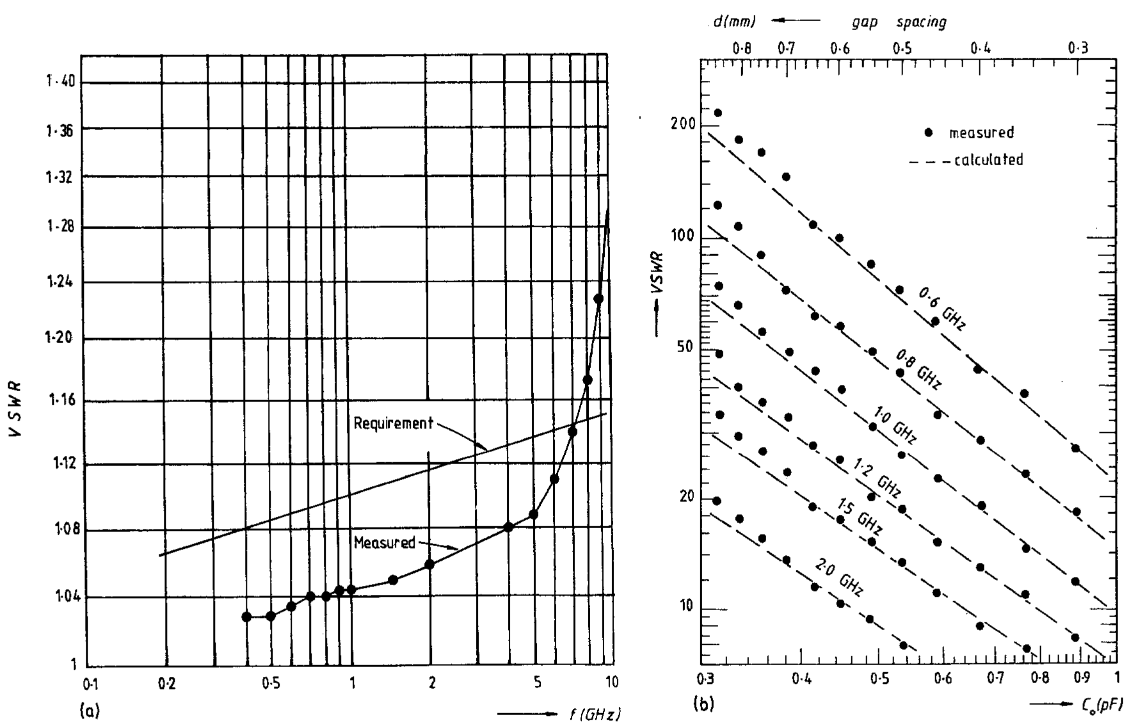


Figure 2 VSWR of the sample holder terminated with 50 Ω at various frequencies for (a) gap closed and (b) different gap spacing.

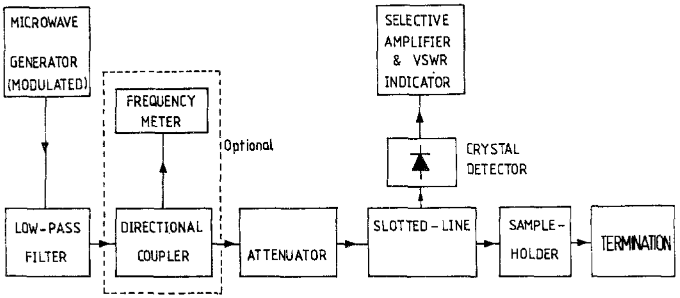


Figure 3 Block diagram of the coaxial line experimental bench.

When the sample holder is terminated with the characteristic impedance, the resulting line load may be approximated to [3, 4],

$$Z_L \approx \frac{1 + j\omega(C_0\epsilon_r^* + C_f)Z_0}{j\omega(C_0\epsilon_r^* + C_f)} \tag{1}$$

where ω is the radian frequency.

The reflection coefficient at A–A' may be expressed in terms of impedances [5] as

$$\Gamma_L = |\Gamma_L|\exp(-j\Theta_L) = \frac{Z_L - Z_0}{Z_L + Z_0} \tag{2}$$

which for air dielectric, i.e. $\epsilon_r^* = 1$, may be written as

$$|\Gamma_a|\exp(-j\Theta_a) = \frac{1}{1 + j2\omega Z_0(C_0 + C_f)} \tag{3}$$

and for a dimensionally equivalent sample of material under test,

$$|\Gamma_s|\exp(-j\Theta_s) = \frac{1}{1 + j2\omega Z_0(C_0\epsilon_r^* + C_f)} \tag{4}$$

Combining Equations 3 and 4 gives

$$\frac{|\Gamma_a|}{|\Gamma_s|}\exp[j(\Theta_s - \Theta_a)] = \frac{1 + 2\omega Z_0 C_0\epsilon'' + j2\omega Z_0 C_0\left(\epsilon' + \frac{C_f}{C_0}\right)}{1 + j2\omega Z_0 C_0\left(1 + \frac{C_f}{C_0}\right)} \tag{5}$$

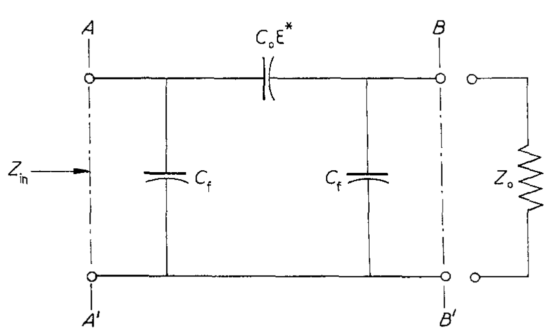


Figure 4 The equivalent circuit for the matched termination method.

On equating the real and imaginary parts and solving explicitly for ϵ' and ϵ'' we finally obtain

$$\epsilon' = \frac{|\Gamma_a|}{|\Gamma_s|}\left\{\left[\left(1 + \frac{C_f}{C_0}\right)^2 + \left(\frac{1}{2\omega Z_0 C_0}\right)^2\right]^{1/2}\sin\Theta_s\right\} - \frac{C_f}{C_0} \tag{6}$$

and

$$\epsilon'' = \frac{|\Gamma_a|}{|\Gamma_s|}\left\{\left[\left(1 + \frac{C_f}{C_0}\right)^2 + \left(\frac{1}{2\omega Z_0 C_0}\right)^2\right]^{1/2}\cos\Theta_s\right\} - \frac{1}{2\omega Z_0 C_0} \tag{7}$$

where $\Theta_s = 2\beta l_s$, l_s being the standing wave minima location with the sample material as the dielectric and $\beta = 2\pi/\lambda_g$, λ_g being the wavelength.

Using Equations 6 and 7 or directly from Equation 4, we can get the following implicit relationship:

$$\frac{\epsilon' + C_f/C_0}{\epsilon'' + 1/2\omega Z_0 C_0} = \tan\Theta_s \tag{8}$$

leading to an approximation

$$\epsilon' \approx \frac{1}{2\omega Z_0 C_0}\tan\Theta_s \tag{9}$$

if

$$\frac{C_f}{C_0} \ll \epsilon'$$

and

$$\epsilon'' \ll \frac{1}{2\omega Z_0 C_0}.$$

The relationships between the reflection coefficient and the dielectric properties of the material under test, for a particular set of dimensions and frequency, are shown in Fig. 5. The plots were obtained using Equations 4 and 8, and require only the experimental values and the reflection coefficient magnitude and its angle to determine ϵ' and $\tan\delta$ for the sample material.

3.2. Measurements

In order to evaluate and determine the validity of the method, known well-documented materials

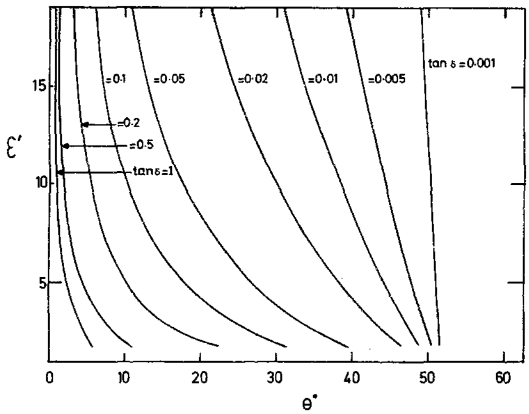


Figure 5 Interdependence of parameters in Equation 8 for 0.5 mm gap at 500 MHz with $C_f/C_0 = 0.112$.

were selected. The choice of a liquid as one of the materials was deliberate, firstly as a new departure and secondly, because of the ease with which it could be placed in the gap of the sample holder and maintained in position by its surface tension. This also gave certain flexibility in the gap spacings and eliminated the lengthy preparation of sample surfaces normally needed with solids.

Measurements were made using pure chlorobenzene for the liquid and a type of perspex PMMA (polymethyl methacrylate) as the representative solid.

The results for chlorobenzene covering the range 0.5 to about 3.0 GHz are given in Fig. 6a and compare favourably with the literature value for ϵ' of 5.7 [8–10]. In the case of PMMA perspex, Fig. 6b, no precise data are available although the value of 2.6 for ϵ' falls within the range of this type of material [16, Moreno-plexiglas, p. 204].

4. The resonant line method

4.1. Theory

If the line is terminated with a reactance instead of the characteristic impedance, Z_0, standing waves will be produced along its length. Such termination can be achieved by using an adjustable lossless stub connected to the line as the load. The

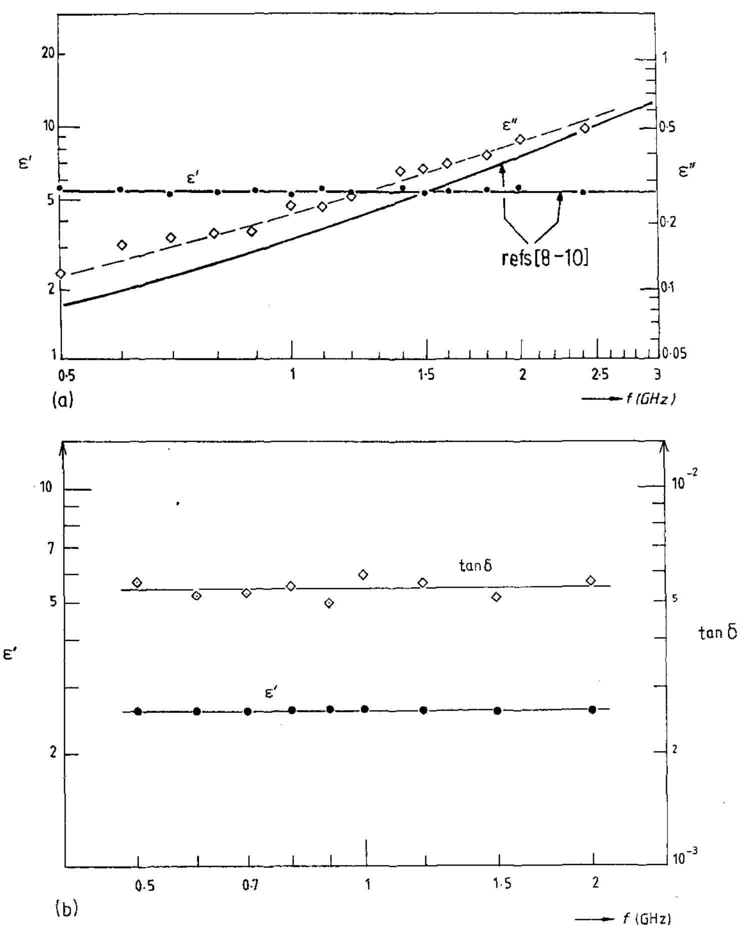


Figure 6 Complex permittivities of (a) chlorobenzene and (b) polymethyl methacrylate (PMMA-perspex) determined using the matched termination method.

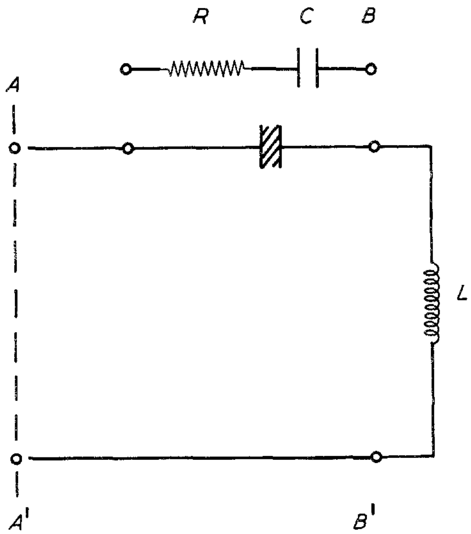


Figure 7 The equivalent circuit for the resonant line method.

line, incorporating the sample holder with air or the material under test in the gap, presents a capacitive impedance (R and C in series). By varying the stub length a maximum reading in the voltage standing wave ratio, VSWR, will be observed when the inductance of the stub cancels the reactive part (C) of the line impedance. This condition of resonance results in a pure resistance (R) which can be related to and deduced from the normalized minimum impedance value given by

$$\frac{R}{Z_0} = \frac{1}{\text{VSWR}_{\text{MAX}}} \tag{10}$$

according to transmission line theory [5]. The equivalent circuit of the arrangement is shown in Fig. 7 and produces the following relationships.

The impedance of the material sample may be written as

$$Z = \frac{1}{j\omega C_0 \epsilon_r^*} \tag{11}$$

where the symbols have the meanings previously defined.

On substitution for the complex permittivity

$$\epsilon_r^* = \epsilon' - j\epsilon''$$

results in

$$Z = \frac{\epsilon'' - j\epsilon'}{\omega C_0(\epsilon'^2 + \epsilon''^2)} \tag{12}$$

which at resonance with the variable terminating stub gives, on equating the real and imaginary parts,

$$\frac{\epsilon''}{\omega C_0(\epsilon'^2 + \epsilon''^2)} = R = \frac{Z_0}{\text{VSWR}_{\text{MAX}}} \tag{13}$$

and

$$\frac{\epsilon'}{\omega C_0(\epsilon'^2 + \epsilon''^2)} = Z_0 \tan \beta l_r \tag{14}$$

where l_r is the stub length at resonance.

Combining Equations 13 and 14 leads to

$$\tan\delta = \frac{\epsilon''}{\epsilon'} = \frac{1}{\text{VSWR}_{\text{MAX}} \tan \beta l_r}. \tag{15}$$

If $\epsilon'' \ll \epsilon'$, approximations may be obtained, namely

$$\epsilon' \approx \frac{1}{\omega C_0 Z_0 \tan \beta l_r} \tag{16}$$

and

$$\tan\delta \approx \epsilon' C_0 \frac{Z_0}{\text{VSWR}_{\text{MAX}}} \tag{17}$$

or

$$\epsilon'' \approx \frac{1}{\omega C_0 Z_0 \text{VSWR}_{\text{MAX}} \tan^2 \beta l_r}. \tag{18}$$

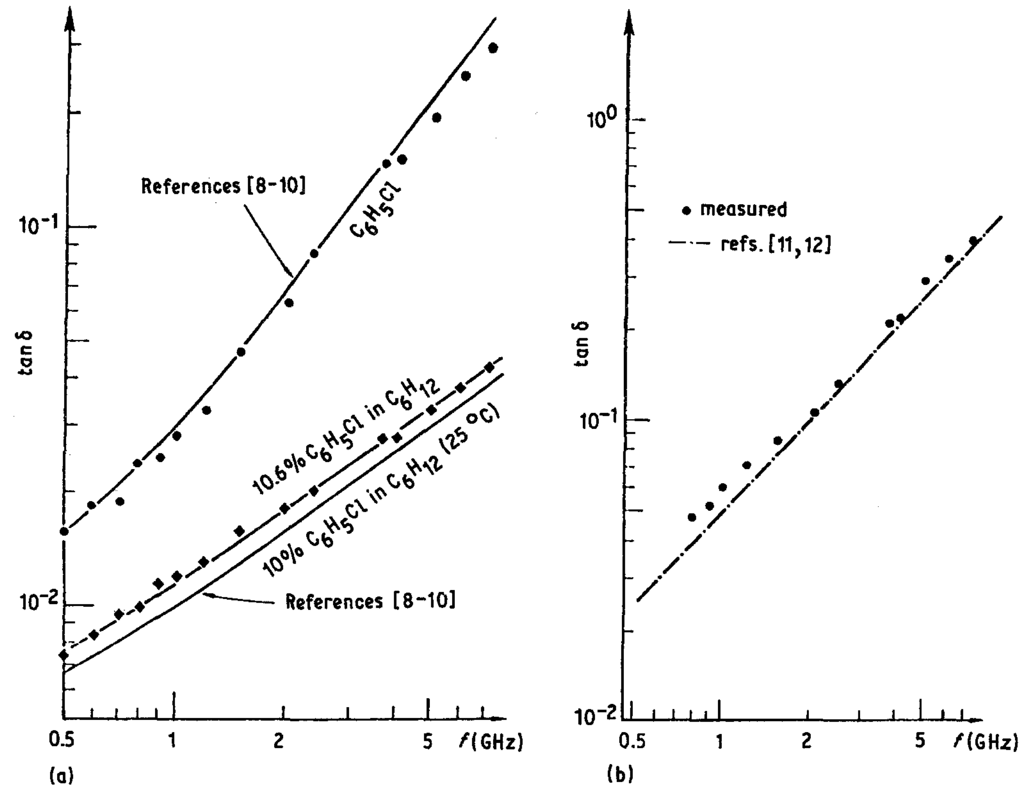
As can be seen from the above, ϵ' may be determined from the resonant condition knowing only the relevant stub length, l_r. However, to determine ϵ'' it is also necessary to know the maximum value of VSWR measured at resonance. The value of $\tan\delta$ is given without any approximations by Equation 15 involving both parameters, VSWR_{MAX} and l_r.

4.2. Measurements

The apparatus arrangement was again as in Fig. 3. This included the sample holder without any modifications. The only difference was that now the line was terminated with an adjustable short circuit.

Solutions of chlorobenzene in cyclohexane and water were used in the measurements as the liquid samples. As previously, these were introduced into the gap of the sample holder using a dropper and the stub was varied until a maximum reading in VSWR was obtained. The values of $\tan\delta$ for pure chlorobenzene, two solutions of chlorobenzene in cyclohexane, and pure distilled water were found in [8–10, 13], and [11, 12], respectively, and plotted for comparison in Figs. 8a and b. The $\tan\delta$'s for PMMA perspex were plotted against values from [14–16] in Fig. 8c.

The changes in the value of maximum VSWR over the frequency range 0.6 to 7.0 GHz for the two liquids are shown in Figs. 9a and b. Typical variations in VSWR plotted against the stub length are illustrated in Figs. 10a to c for three solids, silicon, perspex and teflon.



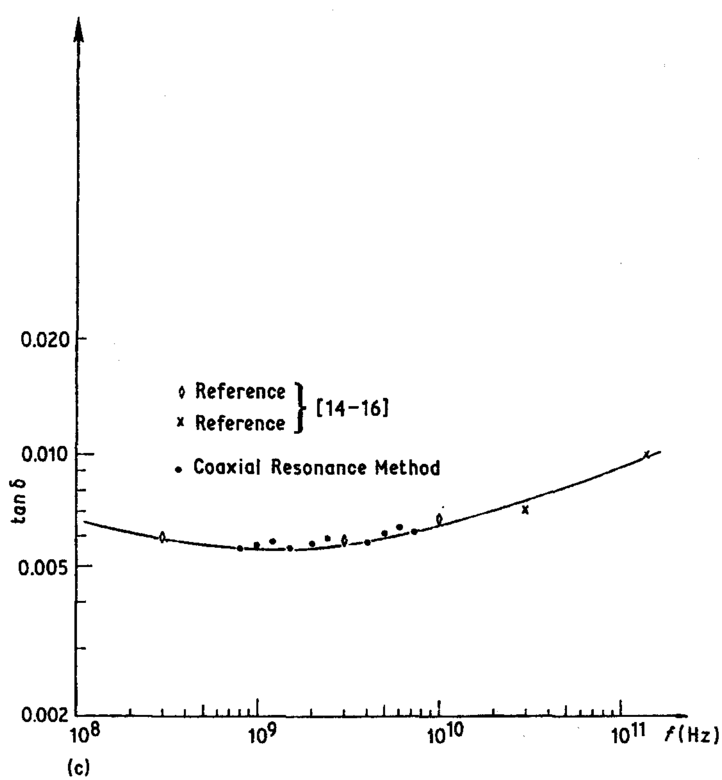


Figure 8 Loss tangent of (a) chlorobenzene and chlorobenzene in cyclohexane solution; (b) pure distilled water; (c) PMMA. Determined using the resonant line method.

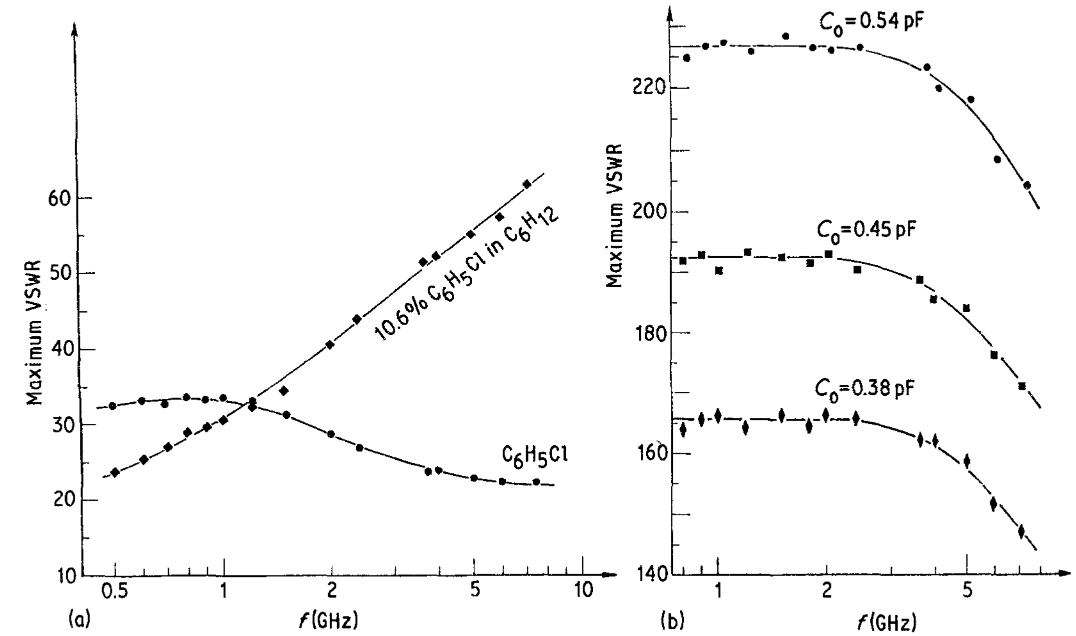


Figure 9 Maximum VSWR readings against frequency for (a) chlorobenzene and chlorobenzene in cyclohexane solution, and (b) pure distilled water for three different gap spacings.

5. Conclusions

The two methods just described, are independent and may be used for complementary measurements or mutual verification of results. These aims can be carried out with ease since the only difference between the two sets of apparatus is in the termination, that is, matched load or variable short circuit stub.

The matched termination method produces reasonably accurate values of ϵ' as can be deduced from Equation 10, and amply supported by the results obtained for chlorobenzene and PMMA perspex shown in Figs. 6a and b, respectively. The values for ϵ'', on the other hand, were not in such a good agreement with the published information for both materials, although still very comparable. It suggests that the method using matched termination may be used with some confidence for finding ϵ'.

The resonant line method was developed primarily for the purpose of determining the values of tan δ in materials having moderate or

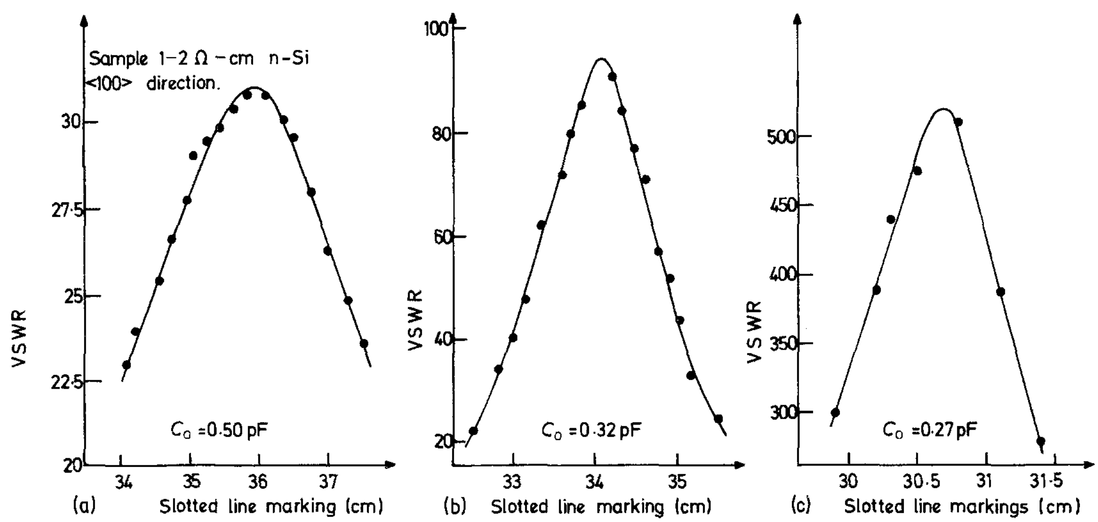


Figure 10 VSWR resonance curves for (a) silicon at 2 GHz, (b) PMMA at 2 GHz, and (c) Teflon at 1 GHz.

low dielectric loss. There are two parameters involved as shown by Equation 17 – the maximum or the resonant value of VSWR and the resultant stub length. Representative examples of VSWR curves in order of increasing magnitude for silicon, PMMA perspex and teflon, respectively are given in Figs. 10a to c. Some difficulty may arise in measuring the high VSWR values expected in the case of good insulators. Highly sensitive quality standing wave meters will normally have the range of 80 dB, 60 dB attenuator plus 20 dB on the meter scale. This would imply a VSWR range of up to 10^4 (20 dB ≡ a factor of 10) which, however, cannot be fully realized in practice due to overloading and noise-at-minima problems. A VSWR of 10^3 is possible with care, allowing $\tan\delta$ of 10^{-3} or less to be measured. There may also be additional difficulty in establishing graphically the actual $VSWR_{MAX}$, as is apparent in Figs. 10b and c for perspex and teflon. If precise measurements are not possible around the peak, extrapolation of the slopes could give an estimate of the maximum value and error involved.

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